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Microscopic Simulations of Shock Wave Propagation in Pseudo-skull

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Abstract: *This paper develops a basic numerical scheme for a pseudo-skull matter and shows how to construct a reliable model. First, we solve one dimensional model and shows the solutions, which we cannot expect before numerical calculation. Next we solve two dimensional model and compare the results of one and two dimensional models. From the comparison, we understand the important feature of the propagation of shock wave, i.e., the shock wave goes to directly on a line without spreading into the other dimension in the matter. This means the one dimensional model is useful, since it can give an essential feature of the shock wave. Finally, we show the energy and momentum conservation low holds while the calculation.*

Keywords *fall accidents , pseudo-skull, shock wave, solution of one and two dimensional models, position, velocity, dummy robot, oscillation.*

1.Introduction

Dead from fall accidents is a severe problem in Japan, especially for senior people. Fall accidents cause often head injury, which is determined by the impulse force and acceleration, and many researches are intended to clarify the relation the force and human injury [1]-[3]. However, the detailed relation between injury and the impulse is quite indefinite because the experiment using human cannot be performed. Therefore, the experiment using a dummy robot similar to human body is conducted, and the result has been analyzed [4] -[7]. The accelerometers have been set to measure in four parts of the body, head, chest, pelvis and neck. These acceleration data is one of the basic data to obtain the relation between the grade of a head shock and cerebral external injury. On the basis of the data, we have deduced the deformation of the head. However, we should check that the deformation is correct and how the micro deformation propagates in the skull. In order to investigate this problem, we analyze a uniform matter. The main subject is to analyze theoretically how micro deformation occurs and propagates in the matter. At first, we adopt a one dimensional model. This model includes essential features which are commonly observed in an actual impulse. Next, we solve a two dimensional model. Comparing the results with the one dimension results, we understand the common features and the specialty of the one dimensional model. These results include new knowledge on the shock wave propagation in a pseudo-skull matter.

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2. One dimensional model

A simple model is considered in one dimension for a step of developing a calculation frame of the propagation of microscopic shock waves. Then we take one dimensional zone, a part of bone, and idealize to discretize in a simple rectangular shapes.

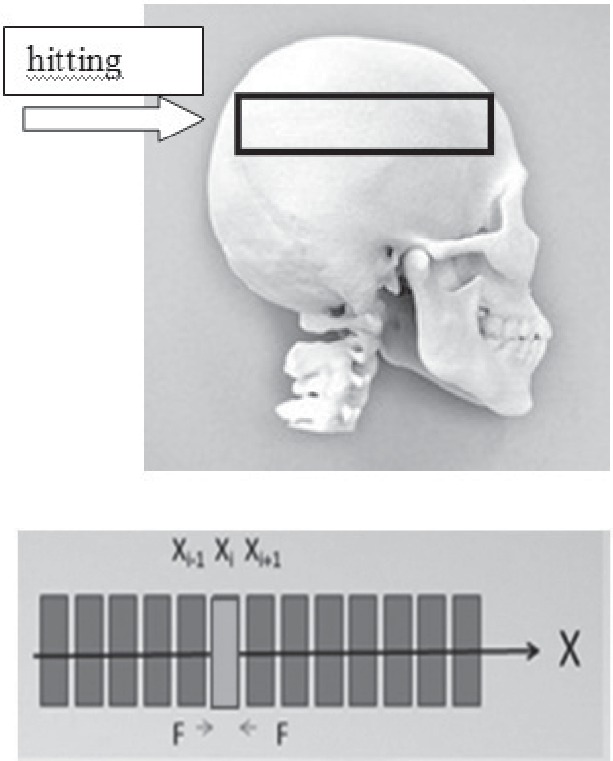

Fig.1 Discretized elements in x- coordinates

Here we assume the part is cubic shape, then each element has the same volume with the aria **S** and the length **L** of one element. Equation of Motion of each element is given as follows in the case of an infinite system with the running number i = infinity,

$$\mathbf{M\ dv_i /dt = K (L -(X_i-X_{i-1})) - K(L -(X_{i+1}-X_i))}$$

Here **t** is time, $\mathbf{X_i}$ is the displacement (deformation) of the i-th element from the original position (natural position) and $\mathbf{v_i}$ is the velocity of the element. The symbol M is the mass of one element, which is given by the product of density **ρ** and the volume **SL. K** is the spring constant and given by **K=ES/L,** where **E** is the young Modulus of the bone.

Our problems in this paper is to analyze more realistic case, then the basic equation has several conditions as bellows.

1) The system is finite, this means the system has both edges, then we should consider the boundary conditions.

2) Initial condition is special. We consider here two cases, i.e., (1) collision case and (2) hitting case, collision case is to collide the floor after fall down, and hitting case is to hit the head by bat or something. For both case, we should adopt a proper initial condition. Thus the most of actual situations is not simple linear case, the solution must be complex due to the boundary condition.

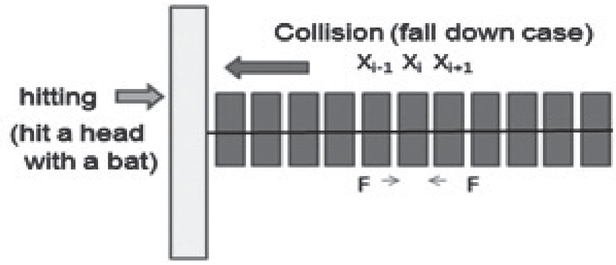

Fig.2 Discretization of the bone in two actual cases

Equation of Motion of the element is different because it has both edges as follows.

For i= 1

$$\mathbf{M\ dv_1 /dt= G (L -X_1) - K(L -(X_2-X_1))}$$

For i= 2 - N-1

$$\mathbf{M\ dv_i /dt= K (L -(X_i-X_{i-1})) - K(L -(X_{i+1}-X_i))}$$

For i= N

$$\mathbf{M\ dv_N /dt= K (L -(X_i-X_{i-1}))}$$

where G is the spring constant between the bone and the wall at the boundary.

3. Method of Numerical Calculation

The simple way to solve the differential equations is to use Eular method, where the time advance in step of h. There are several reasons that Euler's method is not recommended for practical use, among them, (i) the method is not very accurate when compared to other fancier methods with the equivalent step size, the correction is $O(h^2)$, and (ii) neither is it very stable. So we use Runge-Kutta Method, by far the most often used is the classical fourth-order Runge-Kutta formula, the correction is $O(h^5)$, which scheme is as follows. In order to obtain the coordinate X, we add the equation of motion into the following two sets of equations,

$dX_i / dt = V_i$
$= fx (X_{i-1}, X_i, X_{i+1}, V_{i-1}, V_i, V_{i+1})$
dV_i/dt
$= k/M (L -(X_i - X_{i-1})) - K(L -(X_{i+1} - X_i))$
$= fv (X_{i-1}, X_i, X_{i+1}, V_{i-1}, V_i, V_{i+1})$

Setting the time step size h= dt, we calculate trial values at halfway point as follows.

$X_i1 = h\ fx (X_{i-1}, X_i, X_{i+1}, V_{i-1}, V_i, V_{i+1})$
$V_i1 = h\ fv (X_{i-1}, X_i, X_{i+1}, V_{i-1}, V_i, V_{i+1})$
X_i2
$= h\ fx (X_{i-1}+X_{i-1}1/2, X_i+X_i1/2, X_{i+1}+X_{i+1}1/2, V_{i-1}+V_{i-1}1/2, V_i +V_i1/2, V_{i+1}+V_{i+1}1/2)$
V_i2
$= h\ fv (X_{i-1}+X_{i-1}1/2, X_i+X_i1/2, X_{i+1}+X_{i+1}1/2, V_{i-1}+V_{i-1}1/2, V_i +V_i1/2, V_{i+1}+V_{i+1}1/2)$
X_i3
$= h\ fx (X_{i-1}+X_{i-1}2/2, X_i+X_i2/2, X_{i+1}+X_{i+1}2/2, V_{i-1}+V_{i-1}2/2, V_i +V_i2/2, V_{i+1}+V_{i+1}2/2)$
V_i3
$= h\ fv (X_{i-1}+X_{i-1}2/2, X_i+X_i2/2, X_{i+1}+X_{i+1}2/2, V_{i-1}+V_{i-1}2/2, V_i +V_i2/2, V_{i+1}+V_{i+1}2/2)$
$X_i4 = h\ fx (X_{i-1}+X_{i-1}3, X_i+X_i3, X_{i+1}+X_{i+1}3, V_{i-1}+V_{i-1}3, V_i +V_i3, V_{i+1}+V_{i+1}3)$
V_i4
$= h\ fv (X_{i-1}+X_{i-1}3, X_i+X_i3, X_{i+1}+X_{i+1}3, V_{i-1}+V_{i-1}3, V_i +V_i3, V_{i+1}+V_{i+1}3)$

The final values are a weighted average of four values

$X_i = X_i + (X_i 1+2*X_i2+2*X_i3 + X_i4)/6$
$V_i = V_i + (V_i 1+2*V_i2+2*V_i3 + V_i4)/6$

The fourth-order Runge-Kutta method requires four evaluations of the right hand side per step h. For actual calculations, we set the constants in equations. We use the unit of g, mm, micro-second **(μs)** for this system.

Young's modulus E for bone is defined as F/S=Edl/L, and is taken as the same order of the ref. [8].

$$E=10^{10} N/m^2.$$
$$=10^{(-2)} g \cdot mm/\mu s^2/mm^2$$

Thus the spring constant K=E*S/L, the density of the bone $\rho=2g/cm^3$, and we take the length is L=100mm, which is discretized by N=50.

The result has a scaling low in one dimensional model. In the above case, effective dimensionless mass is defined as follows,

$$m= \rho L^2 /ET^2 = w/L\ ET^2$$
$$= 2*10^{-3}*2^2/ 10^{(-2)} = 0.8$$

4. Result of colliding on the floor

Actual numerical solution for m=0.8 in the case of colliding on the floor is shown in the following figure. The time step size 0.1μs is used in these calculations.

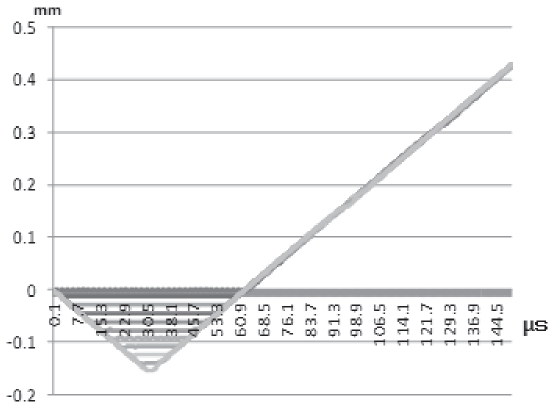

Fig.3 Position of each element. The x-axis represents the time (μs) after colliding, and y-axis is the displacement (mm) of each element. The lines with different color shows the position of the each 10mm element (total is 100mm for 10 elements). The 10 mm element from the hitting edge shows the smallest deformation (blue), then 20 mm element from the hitting edge shows the next one(red), and so on, then 100mm of the other edge moves in the largest deformation(sky blue).

This result shows the motion of each element. The first part, which collides to the floor, proceeds little, and next part goes to more, the last part moves largely (the largest deformation is 0.16mm) then suddenly it returns to opposite way. It is interesting that the other parts follow it together. Next we see the collision time in detail in the following figure. We see the each part is vibrates with a very small amplitude, although it looks like stopping at the same position. This feature is newly found.

We can check the momentum conservation, by calculating the collective motion of the center of mass, V=5m/s, which agrees the initial velocity.

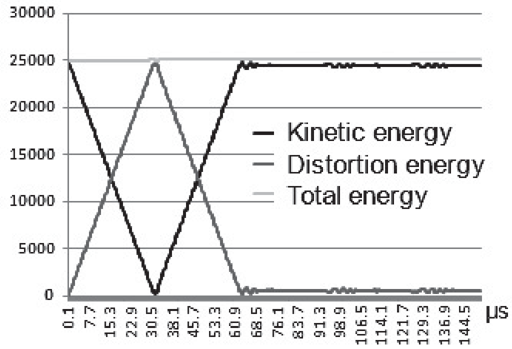

Fig.4 Distortion (Potential), kinetic and total energy. The x-axis represents the time (μs) after colliding, and y-axis is the energy in the unit of g (mm/μs)^2. The green line is the total energy, while the red line is the distortion (potential) energy, and blue line is kinetic energy.

The energy of the system can be calculated directly, kinetic and potential energies are calculated by the position and the velocity of the element, they varies shapely in a linear way as the time development. On the other hand, the total energy of the sum of kinetic and distortion (potential) energy is constant as shown in Fig.4. The constant insures the precision of calculation scheme. In this case, we can observe a little internal energy, which occurs from tiny vibration of each element after the collision.

5. Result of hitting the bone

Hitting the bone from the right hand side has completely different features from the fall down case.

To make comparison easy, we take the same parameters, the spring constant K=E*S/L, the density of the bone ρ=2g/cm^3 , and the length is L=100mm, which is discretized by N=50.

In this case, the initial condition is given as follows. At t=0, the velocity V(1)=50mm/μs for i=1 only, the others are V=0m/s, and the all position is natural one in the bone.

The results are shown in microscopic time scale to 10μs (Fig.5), and in a macroscopic scale of the time from 0 to 0.15 ms (Fig.6). The feature of motion of each element is different from the collision case. In this hitting case, all elements moves with a large vibration. One element moves, then the next one moves after the former one stops almost. It like a successive collision with the next element. The motion of the center of mass is linear of time, and the total momentum is checked to be conserved. The maximum deformation is 0.04 mm, small in this case.

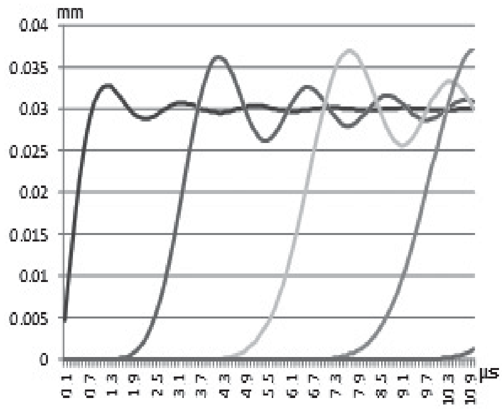

Fig.5 Position of each element from 0 to 10.9μs. The x-axis represents the time (μs) after colliding, and y-axis is the displacement (mm) of 4 elements. The blue line shows 10 mm element from the hitting edge, then red line is 20 mm element from the hitting edge, the green is 30 mm, and the purple is 40 mm from the hitting edge. The position of the other 6 elements is not shown in the figure.

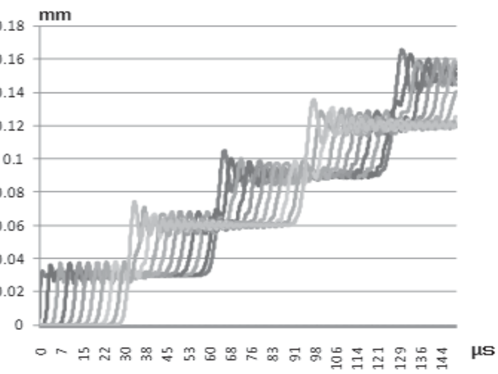

Fig.6 Position of each element from 0 to 144μs. The x-axis represents the time (μs) after colliding, and y-axis is the displacement (mm) of each element. The meaning of the lines with different color is the same as in Fig.2. This figure show the global view of movement.

From the numerical calculation, the speed of CM system is V=1m/s. On the other hand, the theoretical estimation is just V=1m/s from the momentum conservation, which agrees with each other.

The behavior of the directly calculated energy of the system are interesting. The total energy of the sum of kinetic and potential energies is constant as shown in Fig.7, which insures the precision of calculation scheme. On the other hand, the potential and kinetic energy vibrate very quickly around the half value at first, then still keeps half value, and after a some while it vibrate again very quickly around the half value.

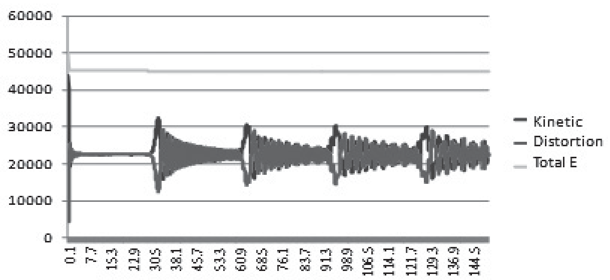

Fig.7 Potential, kinetic and total energy. The x-axis represents the time (μs) after colliding, and y-axis is the energy in the unit of g (mm/μs)^2.
The green line is the total energy, while the red line is the distortion (potential) energy, and blue line is kinetic energy.

6. Two dimensional model

It is important to understand the microscopic mechanism of the deformation of the skull after hitting the head in a more realistic case. In the following sections, we discuss a two dimensional model of simulation on the deformation. We assume a elastic material, and cut it into a finite size to obtain a pile of elastic plates. Then we calculate the deformation of the two dimension for the part of skull in Fig.8. The equation of motion of the plate is coupled differential equations.

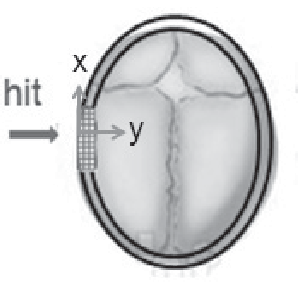

Fig.8 Calculation part and x,y-coordinates. The meshed blue part is the calculation part, which is discretized as 100 (x) by 10 (y) elements, which is 100mm by 10 mm.

For i=1 (outside element),

$$\mathbf{M\ dVx_{1\,j}/dt} = F\mathbf{x}_{i+,j} + F\mathbf{x}_{i,j+} + F\mathbf{x}_{i,,j-}$$

$$\mathbf{M\ dVy_{1\,j}/dt} = F\mathbf{y}_{i+,j} + F\mathbf{y}_{i,j+} + F\mathbf{y}_{i,,j-}$$

For i=2~ N-1 (inner elements),

$$\mathbf{M\ dVx_{1\,j}/dt} = F\mathbf{x}_{i+,j} + F\mathbf{x}_{i-,j} + F\mathbf{x}_{i,j+} + F\mathbf{x}_{i,,j-}$$

$$\mathbf{M\ dVy_{1\,j}/dt} = F\mathbf{y}_{i+,j} + F\mathbf{y}_{I-,j} + F\mathbf{y}_{i,j+} + F\mathbf{y}_{i,,j-}$$

For i=N (inside element),

$$\mathbf{M\ dVx_{1\,j}/dt} = F\mathbf{x}_{i-,j} + F\mathbf{x}_{i,j+} + F\mathbf{x}_{i,,j-}$$

$$\mathbf{M\ dVy_{1\,j}/dt} = F\mathbf{y}_{I-,j} + F\mathbf{y}_{i,j+} + F\mathbf{y}_{i,,j-}$$

where Fx and Fy is the x or y component of the force of from the next element. We assume the force is proportion to the displacement from the natural position, so there is no dependence on the direction.

The numerical calculation of the coordinates, **x** and **y**, and the velocities, **Vx** and **Vy** is performed by Runge-Kutta method on the basis of the above two dimensional equations of motion**,** which are generalized from one dimension case, so it is not described here.

We use the unit of g, mm, micro-second. The natural length of CM of two elements, L = 1 mm. We set 100×10 elements, then the size is 10cm×1cm.

The Young's modulus for bone $E=10^{10}N/m^2$

$=10^{(-2)} g \cdot mm/\mu s^2/mm^2$

For Spring constants K=E*S/L

$=0.01\ g \cdot mm/\mu s^2/mm = 0.01\ g/\mu s^2$

The mass of the one element is mij = density*volume = 0.002 g/mm^3, setting the density $\rho=2g/cm^3$ for bone.

These constants are the same for one dimensional model. Since it is difficult to show the result of calculation, we show only the amplitude of y-direction for selected 13×10 elements.

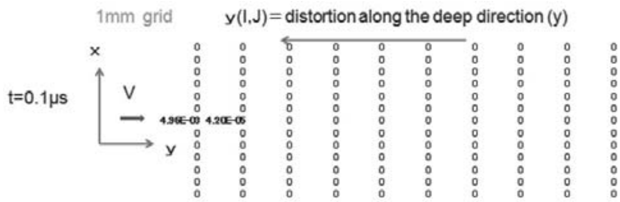

Fig.9 Amplitudes of calculated points at t=0.1μs. This point in the figure are only 13mm(x) by 10mm(y) part of 2-D calculation (100mm by 10 mm). The 0 in the figure indicates the point does not move, while non-zero value shows the point is moving.

The direction x and y are shown in Fig.9. The amplitudes of result are shown on selected times of each several steps. The number 0 shows there are no wave, while the numbers show the non-zero amplitude.

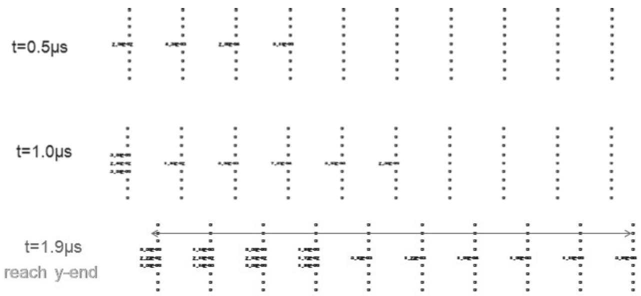

Fig.10 Amplitudes of calculated points till 1.9μs. The meaning of the figure is the same as in Fig.9.

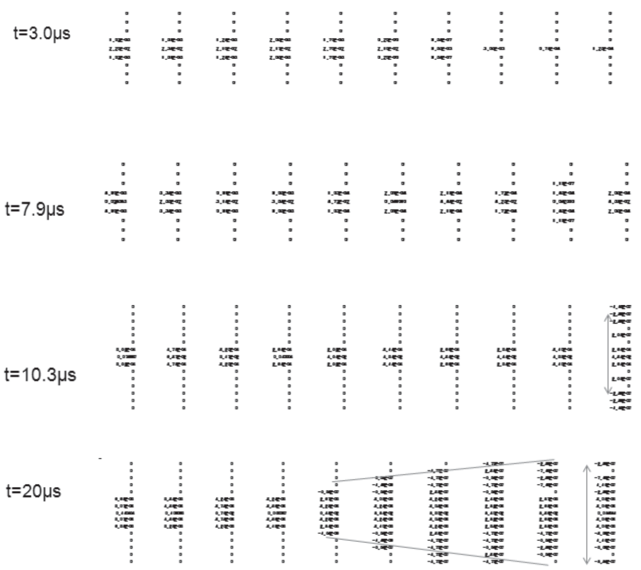

Fig.11 Amplitudes of calculated points till 20μs. The meaning of the figure is the same as in Fig.9.

At t=1.9μs, the shock wave reaches to the y-end (10mm), then the speed of the longitudinal shock wave is deduced as 5263m/s.

At t=10.3μs, the shock wave reaches to the edge of x-direction, then the speed of traverse shock wave is estimated as 2727m/s. These values of the speed of longitudinal and traverse direction are reasonable compared with the experiment.

7. Comparison between one and two dimensional models

In order to compare the results of one and two dimensional models, we adopt the same parameters, i.e., length L=10mm, width 1mm, Young's modulus $E=10^{-2}$, spring constant K=0.01, and mass of the element M=0.002g, and the time step h=0.01μs.

The comparison is done between y-motion of 10 elements on the y-axis (x=0) in the two dimensional model and the y-motion of 10 elements in the one dimensional model. In both models, the initial speed of the outer element is taken as the same value v=50mm/μs.

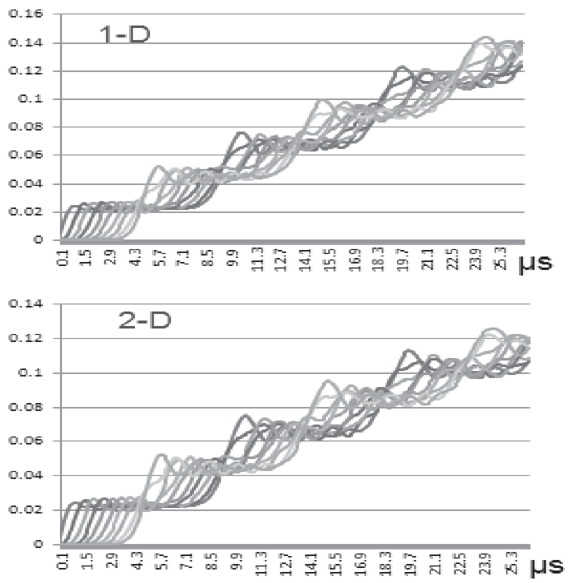

Fig.12 Amplitudes of calculated points till 25.3μs. The upper figure shows 1-D calculation, while the lower is 2-D. The x-axis represents the time (μs) after colliding, and y-axis is the displacement (mm) of each element. The meaning of the lines with different color is the same as in Fig.2.

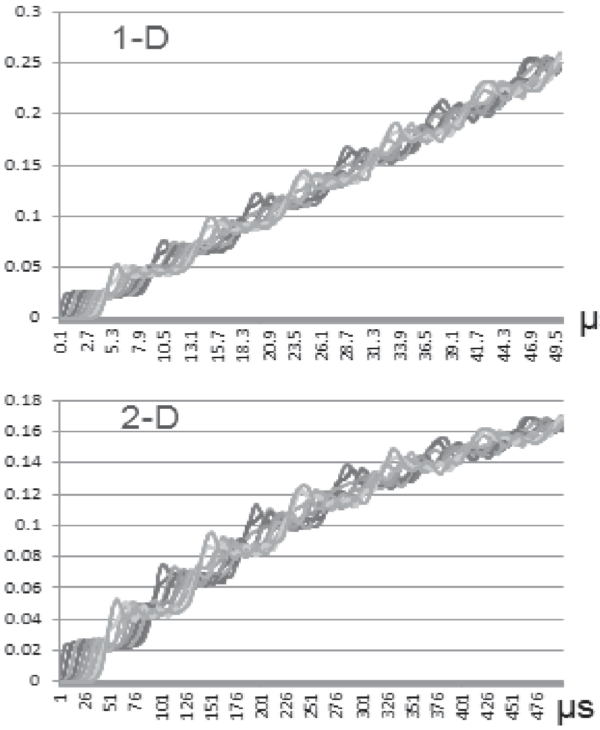

Fig.13 Motions of each element till 47.6μs. The upper figure shows 1-D calculation, while the lower is 2-D. The x-axis represents the time (μs) after colliding, and y-axis is the displacement (mm) of each element. The meaning of the lines with different color is the same as in Fig.2. This figure show the global view of movement.

On this same condition, two models have similar results. Figs 12 and 13 show the motions of each elements in a small range 0 to 25.3μs, and in a long range 0 to 47.6μs. The two dimensional model in the lower figure is similar to the one dimensional model in the upper figure. However, in a longer time, two dimensional model is different from the one dimensional model, they are bent. This is because the energy in the two dimensional model diverse into transverse (x) direction.

Although the energy spreads into the plane, the energy conservation is also checked in the two dimensional model. Fig.14 shows that the kinetic and otential energy vibrate frequently, however, the sum (total energy) is constant, which ensures the precision of the calculation.

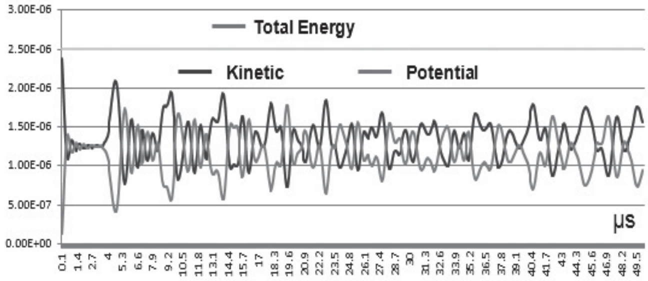

Fig.14 Kinetic and distortion (potential) energy and total energy in two dimensional model. The x-axis represents the time (μs) after colliding, and y-axis is the energy in the unit of g (mm/μs)^2.
The green line is the total energy, while the red line is the distortion (potential) energy, and blue line is kinetic energy.

8.Conclusion

We have performed the analysis of the propagation of the shock wave in the pseudo-skull using one and two dimensional models. The calculations based on the fourth-order Runge-Kutta method work well, we obtained position of each element and their velocities. We found several features of the shock wave, which are new and useful for further progress of the study. The results are different by the boundary conditions. When hitting the edge of bone to the floor, the distortions propagate next to next, then they move together with a almost same velocity. When hitting the edge of bone with a bat, the distortions occur successively, then they move cooperatively with large difference of velocity. The most important feature is that each element is always vibrating in the shock wave propagation, keeping the total momentum and energy. From the comparison between one and two dimensional models, we understand some important facts, i.e., one dimensional model includes essential features. The reason is as follows. 1) The longitudinal shock wave is similar to the two dimensional model. 2) The energy does not diverse into the other directions. 3) The time till the shock wave reaches to the longitudinal edge (2**μs**) is smaller then the time the energy spreads (over 10**μs**). The estimated speed of the longitudinal shock wave is 5263m/s and traverse shock wave is 2727m/s. 4) The speed of the longitudinal shock wave is faster than the transverse, then the shock moves on the straight line. These facts imply that the shock wave goes straight, then it moves deep without energy loss, which may destroy the brain cells in the skull on the line.

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