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The CDMA Product-Form Cellular Telecommunication Network

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Abstract: *This paper proposes queueing network models of cellular mobile communications networks for analyzing the soft capacity peculiar to Code Division Multiple Accesses (CDMA). I formulated these models as Jackson networks with multiple classes and state-dependent transition rates, that is to say, BCMP networks. I derived the product form of equilibrium distribution. As the performance measures, I obtain the explicit probabilities of call losses and handover blockings.*

Keywords: *Mobile telecommunication, BCMP network, Product form, Soft capacity*

1. Introduction

This research includes the evaluation of the traffic in Code Division Multiple Access (CDMA) networks. In mobile communication networks, it is important that we should obtain a standard measure and evaluate the system. As one of the tools, many researchers and engineers have been applied queueing models to solve such a problem. One of the advantages of a cellular mobile communications network is the efficient use of channels since networks reuse these channels in cells, which are sufficiently far away. This efficiency can be further enhanced when multiple calls can be carried using the same channel in a single cell. In this respect, researchers have studied CDMA.

As representative research for mobile communications networks, Everitt and Manfield[1] have studied performance evaluation by using a queueing theory, but their model does not treat as a queueing network model. Since this queueing model, models of mobile communications networks have been studied by several researchers[2][3][4][5][6].

Capacity is a very important parameter for the network operator of any system. The paper by Gilhousen *et.al*[7] is the earliest that considers the capacity of multi-cellular CDMA with emphasis on the analysis of interference from other cells. Unlike in [7] (where they take an equal number of users in each cell), each cell is modeled as an $M/G/\infty$ queue by Viterbi *et.al*[8]. Hence, the number of active calls is a Poisson random variable with mean equal to the traffic offered to each cell. Another paper [9] is an extension to the work in [8]. They also assume an $M/G/\infty$ model for each cell, obtain the distribution function for the interference random variable (considering only distance losses). In [10] they take approximations and bounds for the outage probability and hence obtain traffic capacity by using Asymptotic Expansions and Large Deviations Theory. Many researchers for CDMA capacity evaluation considered multiple access interference and so on. Motivated largely by applications in communications, there are considerable generalizations of the classical Erlang loss models in multi-class, multi-rate, and multi-resource, often referred to as loss networks or resource sharing models, e.g. see Kelly[11], Choudhury, Leung and Whitt[12] and references therein. The basic resource sharing models are continuous time Markov chains with very large state spaces and, because of their special structure; they have the product-form equilibrium distribution[13], just like product-form closed queueing networks. Yoneyama et.al[14][15] map such models on the open Jackson queueing network, which has an infinite number of servers and an infinite capacity, with multiple classes and state-dependent transition rates. The main advantage of such a network is the prod-

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uct-form solution for its stationary distribution[16].

In some researches, a call loss probability for circuit switching systems was the previous fundamental performance measure for CDMA communication networks[17]. It is true that in CDMA communication systems a call loss probability is negligible since all users share a common spectral frequency allocation over the time that they are active and that CDMA communication systems can accept new call as long as there are receiver-processors to serve them. However, blocking in CDMA communication systems would be defined to occur when the interference level, which is due primarily to other user activity, reaches a predetermined level above the background noise level, mainly of thermal origin. To develop a framework to give an approximation can more clarify on the relationships among various variables of the network design. In particular, It is significantly important that the blocking probability and forced termination probability evaluation captured in the form of an approximate probability of deterioration. In addition, the better outlook for performance evaluation is also beneficial for network design.

The trade-offs in CDMA network design have focused on three type's trade-offs: a coverage-capacity trade-off[18], a Quality-of-Service (QoS) trade-off among each calls by soft handover, and a QoS-capacity trade-off by soft capacity. In this study, I focus soft capacity on the trade-off.

The method of the present study is to extend the model[2] of FDMA and TDMA performance evaluation framework for the CDMA networks. The extended portion is that the introduction of soft capacity and soft handover.

The structure of this paper is as follows. In Section 2, I discuss definitions and assumptions for the work in the later sections. In Section 3, I discuss the models and derive the product-form equilibrium distribution from the principle of partial balance by truncating some state space. In order to measure the performance of CDMA networks, in Section 4 I derive the probabilities of call losses and handover blockings. I conclude my work in Section 5.

2. Assumptions and Markov Chain Model

In this research, several areas compose a cell and the whole area is a queue. This model is as same, as occasion that a base station is a queue. A soft handover (multi-way links) area represents physical radio coverage where a call links to more than one cell. The technology of multi-way links allows calls to link not only with a main base station but also with sub-base stations when the power of radiation exceeds the threshold.

I use the following notations mainly based on [2]. To introduce a "class" is for describing the multi-media traffic and for constructing BCMP queueing network[16][19] models, which have quasi-reversibility.

l: the class of a call,

$\alpha_{i(u)}(=\sum_j \lambda_{ij(u)})$: the arrival rate of a class u call at cell i,

$c_{ij(u)}$: the overall arrival rate of a class u call at cell i and area ij including handover,

$\mu(n)$: the service rate of state n,

$\mu_{ij(n)}$: the service rate of a class u call in area ij,

$\nu_{i(n)}$: the overall traffic intensity of a class u call in cell i,

$\nu_{ij(n)}$: the traffic intensity of a class u call in area ij,

π_i : the equilibrium distribution of node i,

$p_{ij(n),kl(v)}$: the routing probability from area ij as a class u call to area kl as a class v call,

C_i: the number of channels in cell i,

W: the set of call classes in the network or the service area,

H_i : the set of cells neighboring cell i,

$P_{i(l)}$: the probability that a class l call exists in cell i,

$n_{ii(u)}$: the number of class u calls in progress in the interior of cell i,

$n_{ij(u)}$: the number of class u calls in progress in the handover area between cells i and j but carried by base station i,

$m_{i(u)}$: the number of class u calls carried by base station i,

$\boldsymbol{m} = (m_1,...,m_N)$: the vector representing the total number of calls in cells, where $m_i = \sum_{u=1}^{I} m_{i(u)}$.

$\boldsymbol{B}$: constraint matrix; a matrix that only contains non-negative entries,

$\boldsymbol{C}$: an appropriate positive vector,

$$\chi := [\boldsymbol{Bm}]_i^{-1}$$

$$U := \{\boldsymbol{m} : \boldsymbol{Bm} \le \boldsymbol{C}\};,$$

$$U_i := \{\boldsymbol{m} : \boldsymbol{B}(\boldsymbol{m} + \boldsymbol{e}_i) \le \boldsymbol{C}\},.$$

$$T_i := \{\boldsymbol{m} : \boldsymbol{Bm} \le \boldsymbol{C}, \boldsymbol{B}(\boldsymbol{m} + \boldsymbol{e}_i) \le \boldsymbol{C}\},$$

$$T_{ij} := \{\boldsymbol{m} : \boldsymbol{B}(\boldsymbol{m} + \boldsymbol{e}_i) \le \boldsymbol{C}, \boldsymbol{B}(\boldsymbol{m} + \boldsymbol{e}_j) \le \boldsymbol{C}\}.$$

For wide types of cellular mobile communications networks, including networks with fixed or dynamic channel allocation, the constraint of the state space is the form of $\boldsymbol{Bm} \le \boldsymbol{C}$. It represents not the stochastic conditions of constraints, but the determined conditions.

Consider a cellular mobile communications network consisting of N cells and I classes of calls. A cell represents physical radio coverage in this research. A handover area represents physical radio coverage where a call links to two cells. There is a capacity constraint on the number of calls that can be carried by base station i.

New class u calls are assumed to be generated by a Poisson process in cell i with rate $\lambda_{ii(u)}$, $i = 1, 2, \ldots, N$, $u = 1, 2, \ldots, I$ and in the handover area between cells i and j with $\lambda_{ij(u)}$, $i, j = 1, 2, \ldots, N$, $u = 1, 2, \ldots, I$. Area ij is different from area ji. For example, $\lambda_{ij(u)}$ is different from $\lambda_{ji(u)}$. In area ij, the call is mainly linked to the base station i. Mobiles carrying a class u call remain in the interior of cell i for an exp($\mu^*_{ii(u)}$) distributed amount of time. Then the class u call proceeds to the handover area with cell j as a class v call with probability $p^*_{i(u),ii(v)}$). In addition, a class u call might also complete in cell i or in the handover area ij. These occur at rates $\mu'_{ii(u)}$ or $\mu'_{ij(u)}$, respectively. Thus, the holding time of a class u call in the interior of cell i is exponentially distributed with mean $1/\mu_{ii(u)}=1/(\mu'_{ii(u)} + \mu^*_{ii(u)})$. With probability $p_{ii(u),0}:= \mu'_{ii(u)}/\mu_{ii(u)}$, the class u call completes, and with probability $p_{ii(u),ij(v)}:= (\mu^*_{ii(u)}/\mu_{ii(u)})\, p^*_{ii(u),ij(v)}$ the class u call proceeds to the handover area with cell j as a class v call. Class u calls remain in the handover area ij for exp($\mu_{ii(u)}$) distributed amount of time. The handover of a class u call as a class v call is attempted with probability $p_{ii(u),ji(v)}$, and a class u call moves or returns to the interior of cell i as a class v call with probability $p_{ij(u),ii(v)}$. Handover policies determine acceptance of an attempted handover.

Because of the cellular characteristics of mobile communication networks, these networks have been modeled using queueing networks; a cell is a queue. A major difference between mobile networks and queueing networks is that in cellular communication networks the holding time of a call relate to the call, whereas the service time of customers in queueing networks relate to the queue. When a call moves from one cell to another, the residual call holding time is an important parameter.

Only under the assumption of exponentially distributed call holding times is a standard queueing network model justified, because the call holding time can be resample upon a handover.

The assumption of exponentially distributed holding times allows the network to be modeled as a continuous-time Markov chain $X=(X(t),t\ge 0)$ that records the number of calls in progress in the areas of cells. The state of this Markov chain is represented as a vector $\boldsymbol{n}=(\boldsymbol{n}_{ii}, \boldsymbol{n}_{ij}, j\in H_i, i=1,\ldots,N)$, where $\boldsymbol{n}_{ii} = (n_{ii(1)},\ldots,n_{ii(I)})$ and $\boldsymbol{n}_{ij} = (n_{ij(1)},\ldots,n_{ij(I)})$.

I assume that the transition of state $\boldsymbol{X}$ is Markovian and the behavior of soft capacity is Markovian. I also assume that the transition of calls both from a one-way link to two-way links and from two-way links to a one-way link is Markovian. I assume that the Markov chain is irreducible at its state space $S = \{\boldsymbol{m}: \boldsymbol{Bm} \le \boldsymbol{C}\}$. Therefore, the Markov chain constitutes the model of CDMA networks.

The Markov chain described above is non-reversible, rather than the reversible. It is because there are a call blocking and a forced termination. It is possible for a reversible Markov chain to derive the product-form solutions from the equilibrium distribution on assuming a steady state. However, it is not possible for the non-reversible under normal conditions. In this research, I derive the performance measures by approximating the non-reversible Markov chain by the reversible.

I assume the following conditions, so that I obtain equations on the traffic flow among areas for $u = 1,\ldots,I$.

$$p_{ii(u),0} + \sum_{j \in H_i} \sum_{v=1}^{I} p_{ii(u),ij(v)} = 1,$$
$$p_{ii(u),0} + \sum_{v=1}^{I} (p_{ij(u),ii(v)} + p_{ij(u),ji(v)}) = 1. \quad (1)$$

The above equations are the normalizing conditions with respect to routing probabilities to obtain the product form distributions for queueing networks.

3. The Model with Soft Capacity

Soft capacity is one of the features of CDMA. Soft capacity is the way of communication where many calls can link at the same time by degrading the bit rate when the number of calls exceeds the threshold. Hence, the maximum number of carried calls is not necessarily fixed. Soft capacity enables a trade off between transmission quality and the number of calls to realize smoothly. Soft capacity provides an additional call in an overloaded cell at the cost of QoS for all calls in that cell.

First, I consider the model with state-dependent transitions. I assume that the capacity of the base station in each cell is sufficient to satisfy $C_i = \infty$. The model is the BCMP network. It has a product-form solution since the state-dependent rate is special as described below.

I assume that any call that enters a cell will eventually leave, and the following traffic equations have to be satisfied for $u = 1, 2, \ldots, I$.

$$c_{ii(u)} = \lambda_{ii(u)} + \sum_{j \in H_i} \sum_{v=1}^{I} c_{ij(v)} p_{ij(v),ii(u)}, \quad (2)$$

$$c_{ii(u)} = \lambda_{ii(u)} + \sum_{v=1}^{I} c_{ii(v)} p_{ii(v),ij(u)} + \sum_{v=1}^{I} c_{ji(v)} p_{ji(v),ij(u)}, \quad (3)$$

Note that the above traffic equations have solutions the $c_{ii(u)}$'s and $c_{ij(u)}$'s with some constraint conditions.

In this model, for $I = 1, 2, \ldots, N$, and $u = 1, 2, \ldots, I$, the rate $\mu_{ii(u)}$ when the state of the network is $\boldsymbol{n}$ is replaced by

$$\mu_{ii(u)}(\boldsymbol{n}) = \mu_{ii(u)} \frac{\Psi(\boldsymbol{n} - \boldsymbol{e}_{ii(u)})}{\Phi(\boldsymbol{n})}, \quad (4)$$

and the rate $\lambda_{ii(u)}$ when the state of the network is $\boldsymbol{n}$ is replaced by

$$\lambda_{ii(u)}(\boldsymbol{n}) = \lambda_{ii(u)} \frac{\Psi(\boldsymbol{n})}{\Phi(\boldsymbol{n})}, \quad (5)$$

where Ψ and Φ are arbitrary nonnegative and positive functions, respectively. They represent the coefficient of the capacity. If I set the coefficient of the capacity properly, I can represent various servers such as diverse service disciplines.

Also, for $j \in H_i$, $i=1, 2, \ldots, N$, $u=1, 2, \ldots, I$, the rate $\mu_{ij(u)}$ is replaced by

$$\mu_{ij(u)}(\boldsymbol{n}) = \mu_{ij(u)} \frac{\Psi(\boldsymbol{n} - \boldsymbol{e}_{ij(u)})}{\Phi(\boldsymbol{n})}, \quad (6)$$

and the rate $\lambda_{ij(u)}$ is replaced by

$$\lambda_{ij(u)}(\boldsymbol{n}) = \lambda_{ij(u)} \frac{\Psi(\boldsymbol{n})}{\Phi(\boldsymbol{n})}, \quad (7)$$

when the state of the network is $\boldsymbol{n}$. The research [15] presents these state-dependent rates.

The following theorem presents the product form result for a mobile communication network with multiple classes and state-dependent transitions.

Theorem 1 (*State-dependent Transition*)
Let $\{c_{ii(u)}, c_{ij(u)}, j \in H_i, i=1,\ldots,N, u=1,\ldots,I\}$ be the solution of the traffic equations (2) *and* (3). *Then, the equilibrium distribution for a cellular mobile communications network with multiple classes, state-dependent transition rates and infinite capacity is consistent with*

$$\pi(\boldsymbol{n}) = G^{-1}\Phi(\boldsymbol{n}) \prod_{i=1}^{N} \prod_{j \in H_i} \frac{1}{n_{ii(1)}!, \ldots, n_{ii(I)}!} \frac{1}{n_{ij(1)}!, \ldots, n_{ij(I)}!} \times \prod_{v=1}^{I} \left(\frac{c_{ii(v)}}{\mu_{ii(v)}} \right)^{n_{ii(v)}} \left(\frac{c_{ij(v)}}{\mu_{ij(v)}} \right)^{n_{ij(v)}}, \quad (8)$$

$$G = \sum_{\boldsymbol{n}} \Phi(\boldsymbol{n}) \prod_{i=1}^{N} \prod_{j \in H_i} \frac{1}{n_{ii(1)}!, \ldots, n_{ii(I)}!} \frac{1}{n_{ij(1)}!, \ldots, n_{ij(I)}!} \times \prod_{v=1}^{I} \left(\frac{c_{ii(v)}}{\mu_{ii(v)}} \right)^{n_{ii(v)}} \left(\frac{c_{ij(v)}}{\mu_{ij(v)}} \right)^{n_{ij(v)}} 1(\boldsymbol{n} \in S). \quad (9)$$

In Theorem 1 there are no lost calls since the model has infinite capacity. This model is the BCMP network whose capacity is infinite. There are no lost calls or no forced terminations in that phase. However, it is possible to include the lost call and the forced termination of soft capacity model by truncating the state space using the constraint matrix. Later in the performance evaluation of the soft capacity model in Theorem 2, I truncate the state space by S'.

Theorem 2 (*Multi-Class Soft Capacity*)
In theorem 1 let the constraint of the capacity

$$\Phi(\boldsymbol{n}) = \prod_{i=1}^{N} \left(\frac{1}{\chi} \right)^{\max(0, n_{ii} + \sum_{j \in H_i} n_{ij} - C_i)} . \tag{10}$$

Then I get stationary distributions,

$$\pi(\boldsymbol{n}) = G^{-1} \prod_{i=1}^{N} \left(\frac{1}{\chi} \right)^{\max(0, n_{ii} + \sum_{j \in H_i} n_{ij} - C_i)}$$

$$\times \prod_{j \in H_i} \frac{1}{n_{ii(1)}!, \ldots, n_{ii(I)}!} \frac{1}{n_{ij(1)}!, \ldots, n_{ij(I)}!} \tag{11}$$

$$\times \prod_{v=1}^{I} \left(\frac{c_{ii(v)}}{\mu_{ii(v)}} \right)^{n_{ii(v)}} \left(\frac{c_{ij(v)}}{\mu_{ij(v)}} \right)^{n_{ij(v)}} ,$$

where G is the normalizing constant.

In Theorem 2, the rate of degradation of QoS does not depend on the state of the network. The next is a model that depends on the state of the network.

Theorem 3 (*State-dependent degraded QoS*)

Let $\{c_{ii(u)}, c_{ij(u)}\, j \in H_i, i=1,\ldots,N, u=1,\ldots,I\}$ *be the solution of the traffic equations and the constraint of the capacity be* Ξ. *Then, the equilibrium distribution for a cellular mobile communications network with multiple classes and state-dependent degraded QoS is*

$$\pi(\boldsymbol{n}) =$$

$$G^{-1} \prod_{i=1}^{N} \Big\{ 1\Big(n_{ii} + \textstyle\sum_{j \in H_i} n_{ij} \le C\Big)$$

$$+ \left(\frac{1}{c_i} \right)^{\left(n_{ii} + \sum_{j \in H_i} n_{ij} - C_i\right)} \left(\frac{\left(n_{ii} + \sum_{j \in H_j} n_{ij}\right)}{c_i!} \right) 1\Big(n_{ii} + \textstyle\sum_{j \in H_j} n_{ij} > C_i\Big) \Big\}$$

$$\times \prod_{j \in H_i} \frac{1}{n_{ii(1)}!, \ldots, n_{ii(I)}!} \frac{1}{n_{ij(1)}!, \ldots, n_{ij(I)}!} \prod_{v=1}^{I} \left(\frac{c_{ii(v)}}{\mu_{ii(v)}} \right)^{n_{ii(v)}} \left(\frac{c_{ij(v)}}{\mu_{ij(v)}} \right)^{n_{ij(v)}} , \tag{12}$$

where G is the normalizing constant.

Theorem 3 is a special case of the state-dependent transition model in Theorem 2.

4. Performance Measure as the Blocking Probability

The given soft-capacity model, which is truncating the state space, approximates the model that a call loss and a forced termination happen. I regard the derived probabilities in the approximate model as approximations of probabilities of call losses and handover blockings in the original model.

Handover is the phenomenon that a mobile terminal moves from the radio coverage of one cell to that of another one. However, if there is no available channel in the new cell, the call is lost. This is so-called *"handover blocking"*. As the performance measures, I treat the call loss probability for new calls and the blocking probability for a handover. The probability that a handover from cell i to cell j is blocked; that is, P_{ij}^{H} equals the fraction of handovers from cell i to cell j that are lost due to capacity constraints in cell j. This blocking probability is the conditional probability that a call attempted to handover from cell i to cell j and lost when such a call attempt to handover. Forced termination occurs for the call from the interior area to the handover area when the call cannot catch any channel in the handover area.

The probability-flux of handover attempts from cell i to cell j, say $F_{ij(u,v)}^{H}$, is the sum of the rates of the successes of handovers and of the terminations of calls in the handover area.

$$F_{ij(u,v)}^{H} = \sum_{\boldsymbol{n} \in S'} \pi(\boldsymbol{n}) \{ q(\boldsymbol{n}, -\boldsymbol{e}_{ij}) + q(\boldsymbol{n}, -\boldsymbol{e}_{ij} + \boldsymbol{e}_{ji}) \}$$

$$= \sum_{\boldsymbol{n} \in S'} \prod_{v=1}^{I} \frac{\Phi(\boldsymbol{n})}{G} \prod_{i=1}^{N} \left(\frac{c_{ii(v)}}{\mu_{ii(v)}} \right)^{n_{ii(v)}} \frac{1}{n_{ii(1)}!, \ldots, n_{ii(I)}!}$$

$$\times \prod_{j \in H_i}^{N} \left(\frac{c_{ij(v)}}{\mu_{ij(v)}} \right)^{n_{ij(v)}} \frac{1}{n_{ij(1)}!, \ldots, n_{ij(I)}!}$$

$$\times 1(\boldsymbol{n} \in S')(n_{ii(u)} \mu_{ii(u)} p_{ij(u),0} + n_{ij(u)} \mu_{ij(u)} p_{ij(u),ji(v)})$$

$$= \sum_{\boldsymbol{n} \in S'} \frac{\Phi(\boldsymbol{n})}{G} \prod_{i=1}^{N} \left(\frac{v_{ii(v)}^{n_{ii}} v_{ij(v)}^{n_{ij}}}{n_{ii(v)}^{n_{ii}}! \, n_{ij(v)}^{n_{ij}}!} \right)$$

$$\times (n_{ii(u)} \mu_{ii(u)} p_{ij(u),0} + n_{ij(u)} \mu_{ij(u)} p_{ij(u),ji(v)}) 1(\boldsymbol{n} \in S')$$

$$= \sum_{\boldsymbol{n} \in S'} \frac{\Phi(\boldsymbol{n})}{G} \sum_{\boldsymbol{n}} \prod_{i=1}^{N} \prod_{j \in H_i} \left(\frac{v_{ii(v)}^{n_{ii}} v_{ij(v)}^{n_{ij}}}{n_{ii(v)}^{n_{ii}}! \, n_{ij(v)}^{n_{ij}}!} \right)$$

$$\times 1(\boldsymbol{B}(\boldsymbol{m} + e_i) \le \boldsymbol{C})(n_{ii(u)} \mu_{ii(u)} p_{ij(u),0} + n_{ij(u)} \mu_{ij(u)} p_{ij(u),ji(v)})$$

$$= \sum_{\boldsymbol{n} \in S'} \frac{v_{k(v)}^{m_k}}{m_k} \frac{\Phi(\boldsymbol{n})}{G} (n_{ii(u)} \mu_{ii(u)} p_{ij(u),0} + n_{ij(u)} \mu_{ij(u)} p_{ij(u),ji(v)})$$

$$\times 1(\boldsymbol{B}(\boldsymbol{m} + e_i) \le \boldsymbol{C}) \cdot \tag{13}$$

The probability flux of lost handover attempts from cell i to cell j, that is to say $F_{ij(u,v)}^{L}$, is the probability of the overflow in cell j under the increase in the number of calls in cell i.

$$F^L_{ij(u,v)} = \sum_{n \in S'} \pi(\boldsymbol{n})\{q(\boldsymbol{n}, -\boldsymbol{e}_{ij})\}$$

$$= \sum_{m} \prod_{k=1}^{N} \frac{v_{k(v)}^{m_k}}{m_k!} \frac{\Phi(\boldsymbol{n})}{G} n_{ii(u)} \mu_{ii(u)} p_{ij(u),0} \quad (14)$$

$$\times 1(\boldsymbol{B}(\boldsymbol{m}+\boldsymbol{e}_i) \le C) 1(\boldsymbol{B}(\boldsymbol{m}+\boldsymbol{e}_j) > C).$$

The handover blocking probability P_{ij}^{H} equals the ratio of the above two expressions:

$$P_{ij}^{H} = \frac{F^L_{ij(u,v)}}{F^H_{ij(u,v)}}$$

$$= \frac{n_{ii(u)} \mu_{ii(u)} p_{ij(u),0}}{n_{ii(u)} \mu_{ii(u)} p_{ij(u),0} + n_{ij(u)ji(v)} \mu_{ii} p_{ij(u),ji(v)}} \quad (15)$$

$$\times \left(\sum_{\boldsymbol{m} \in T_{ij}} \frac{v_{i(v)}^{m_i}}{m_1!} \right) \Big/ \left(\sum_{\boldsymbol{m} \in U_i} \frac{v_{i(v)}^{m_i}}{m_1!} \right).$$

I can derive the handover blocking probability since the equilibrium distribution is product form. In particular, when the probability of the state space outside the truncated state is negligible,

$$\pi(\boldsymbol{n}) \approx \pi(\boldsymbol{m}) = \prod_{k=1}^{N} \frac{v_{k(v)}^{m_k}}{m_k!} 1(\boldsymbol{B}\boldsymbol{m} \le C), \quad (16)$$

and I can easily evaluate the performance measure by multiplying the same number $\exp\left(-\sum_{k=1}^{N} v_{k(v)}\right)$ for both the numerator and the denominator of (16). This gives

$$\frac{\sum_{\boldsymbol{m} \in T_{ij}} \pi(\boldsymbol{m})}{\sum_{\boldsymbol{m} \in U_i} \pi(\boldsymbol{m})} = \left(\sum_{\boldsymbol{m} \in T_{ij}} \prod_{k=1}^{N} \frac{v_{k(v)}^{m_k} e^{-v_{k(v)}}}{m_k!} \right) \Big/ \left(\sum_{\boldsymbol{m} \in U_i} \prod_{k=1}^{N} \frac{v_{k(v)}^{m_k} e^{-v_{k(v)}}}{m_k!} \right), \quad (17)$$

The right-hand side of (17) is the ratio of two Poisson probabilities. To put it more formally, let $\boldsymbol{X}(v)$ be an N-dimensional random variable, having independent marginal, that are Poisson with means $v_{k(l)}$, $k = 1,\ldots,N$. The performance measure equals $P(\boldsymbol{X}(\boldsymbol{v}) \in T)/P(\boldsymbol{X}(\boldsymbol{v}) \in U)$, and I can evaluate by estimating both the numerator and the denominator. I set U as all of the events and T as the particular events.

In the same way, a call loss probability for a new call P_i^I, which represents the conditional probability that losses occur when a new call arrives, is as follows.

$$P_i^I = \sum_{n:(\boldsymbol{m}+\boldsymbol{e}_i) \le C} \pi(\boldsymbol{n}),$$

$$= \left(\sum_{\boldsymbol{m} \in T_i} \prod_{i=1}^{N} \frac{v_{i(v)}^{m_i} e^{-v_{i(v)}}}{m_i!} \right) \Big/ \left(\sum_{\boldsymbol{m} \in U} \prod_{i=1}^{N} \frac{v_{i(v)}^{m_i} e^{-v_{i(v)}}}{m_i!} \right), \quad (18)$$

P_i^I and P_{ij}^H are obtained only through $\boldsymbol{m}$ since I select the performance measures which depend only on the state $\boldsymbol{m}$.

5. Conclusion

The fundamentals of this research are conventional telecommunication traffic theories, which can develop efficient modeling for empowering capacity calculations in CDMA cellular networks. In the formulation, I introduced the discrete-state Markov chain to take the number of call in each cell. QoS degradation due to increased customer calls, the state probability distribution, QoS explicitly asked as a product of the steady-state probability distribution representing the indicator function deterioration. Next, in order to express the movement of customers in cellular mobile networks in the queue, I give the routing probability for each cell (node in the context of the queueing model), and looking for steady-state probability distribution, with truncating a capacity as a threshold. I eventually develop a framework to evaluate the performance of the mobile communication network.

The proposed models are useful analysis for planning load balancing as well as quantitative analysis by investigating statistically the behavior of call.

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